UDC 539.3:534. 231.1

DIFFRACTION OF A PLANE WAVE BY A PLATE REINFORCED BY A PROTRUDING STIFFENING RIB

PMM Vol.42, №3, 1978, pp.486-493

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(Received March 10, 1977)

The diffraction of a plane wave by a plate stiffened by a rib of small-wave dimension is considered. The problem is reduced to an integral equation of the second kind for the pressure at the rib surface. Principles of contractive mappings are applied to the equation in the long-wave range. Approximate formulas are derived for the directivity pattern of the field dispersed in the fluid and for the pressure at the rib surface.

The diffraction of sound at a plate with a stiffening rib was investigated in [1, 2], and the effect of the rib wave properties on the plate emission was considered in [3]. The feasibility of compensating the emission of a plate stiffened by a rib by the application of additional forces was discussed in [4]. A common simplifying feature of all these investigations is the neglect of sound reflection by the rib surface, i.e. it was assumed that the rib affects only the conditions under which oscillations of the reinforced plate occur.

In the present investigation the effect of the rib reflecting surface on the diffraction field is taken into account. The directivity pattern of the field generated by the incidence of a plane wave on the plate and emitted into the fluid and the pressure distribution on the rib surface are determined. Effects of plate thickness, rib height, and of the plane wave incidence angle on power emission to the fluid is investigated. Limits of applicability of the approximate analysis of diffraction processes without allowance for the reflection of sound from the rib surface are discussed.

1. Let a plane wave of pressure $P_0 = \exp(-i\omega t + ikc_0x - iks_0y)$ impinge on plate $\{-\infty < x < \infty,\ y = 0\}$ with rib $\{x = 0,\ 0 < y < h\}$. Here $c_0 = \cos\varphi_0$, $s_0 = \sin\varphi_0$, φ_0 is the incidence angle read from the semiaxis $x > 0$ and k is the wave number in the fluid. The problem is assumed plane, and the dependence on time $\exp(-i\omega t)$ (ω is the oscillation frequency) is neglected. Pressure $P(x, y)$ $(y > 0)$ in the fluid satisfies the Helmholtz equation with the following boundary conditions at the plate [1, 5]:

$$LP(x,0) = \left[\left(\frac{\partial^4}{\partial x^4} - k_0^4\right)\frac{\partial}{\partial y} + q^5\right]P(x,0) = B\delta(x) + C\delta'(x) \qquad (1.1)$$

$$q^5 = \rho_0\omega^2/D \qquad (-\infty < x < \infty)$$

where k_0 is the wave number of the plate flexural oscillations, ρ_0 is the fluid density, D is the plate cylindrical stiffness, $\delta(x)$ is the delta function [6], and B and C are the so-called boundary-contact constants determined by conditions at the rib-to-plate interface. In what follows the wave dimensions of the rib are assumed small and consequently its wave properties may be disregarded, i.e. it can be assumed that the rib moves as a whole, performing longitudinal oscillations along the y-axis and rotations about an axis passing through the coordinate origin and normal to the xy -plane. Using the equations of motion of the rib subjected to a discontinuity of the shear stress at its joint to the plate and, also, of the bending moment, and the difference of pressure on both sides of the rib, we obtain the following boundary-contact conditions:

$$Z_1 P_y(0,0) = [P_{yxxx}(0,0)] \tag{1.2}$$

$$-Z_2 P_{yx}(0,0) = [P_{yxx}(0,0)] + q^5 \int_0^h s\,[P(0,s)]\,ds$$

$$Z_1 = \rho_1 h H_1 \omega^2 / D, \quad Z_2 = \rho_1 h^3 H_1 \omega^2 / (3D)$$

The symbol $[f(0)]$ denotes the discontinuity of function $f(x)$ at transition through point $x = 0$. In (1.2) Z_1 and Z_2 are rib impedances [1,3], ρ_1 is the density of the rib material, and H_1 is its thickness. The boundary condition at the surface of the oscillating rib is derived from the condition of bonding and is of the form

$$P_x(0,y) = -yP_{yx}(0,0) \qquad (0<y<h) \tag{1.3}$$

We separate from the complete field the incident wave and the wave reflected by the homogeneous plate (R are coefficients of reflection)

$$P(x,y) = \exp(ikc_0x - iks_0y) + R\exp(ikc_0x + iks_0y) + Q(x,y)$$

$$R = [(c_0^4 - a)\,is_0 - b]\,[(c_0^4 - a)\,is_0 + b]^{-1}, \quad a = k_0^4 / k^4, \, b = q^5 / k^5$$

The dissipated field is constructed in conformity with the principle of limit absorption, and must satisfy the Meixner condition "at the rib" [7]. In the zero approximation, in which diffraction at the rib surface is neglected, the field determined by the method described in [1] is

$$Q_0(x,y) = -iks_0(1-R)\int_{-\infty}^{\infty} \exp[i\lambda x - \gamma(\lambda)y] \times \tag{1.4}$$

$$\left(\frac{1}{J_{0,1} - 2\pi/Z_1} + \frac{\lambda k c_0}{J_{2,1} - 2\pi/Z_2}\right)\frac{d\lambda}{L(\lambda)}, \qquad J_{m,n} = \int_{-\infty}^{\infty} \frac{\lambda^m \gamma^n(\lambda)}{L(\lambda)}\,d\lambda$$

$$L(\lambda) = (\lambda^4 - k_0^4)\,\gamma(\lambda) - q^5, \quad \gamma(\lambda) = \sqrt{\lambda^2 - k^2}$$

The branch of radical $\gamma(\lambda)$ and the integration contour are selected in conformity with the principle of limit absorption [5]. When determining the field Q_0, the integral term in Eq. (1.2) and condition (1.3) are neglected. Such simplification is a feature of all preceding investigations. One of the aims of the present paper is to examine its validity.

2. The field $w = Q - Q_0$ related to the diffraction at the rib surface satisifes the Helmholtz equation with boundary condition (1.1) on the plate, condition

$$w_x(0,y) + yw_{yx}(0,0) = g(y) \quad (0<y<h) \tag{2.1}$$

$$g(y) = -kc_0\left\{i\left[\exp(-iks_0y) + R\exp(iks_0y)\right] + ks_0\frac{2b}{l_0}\times\right. \tag{2.1}$$

$$\left.\frac{1}{J_{2,1} - 2\pi/Z_2}\left[\int_{-\infty}^{\infty}\frac{\lambda^2}{L(\lambda)}\exp\left(-\gamma(\lambda)y\right)d\lambda - y\frac{2\pi}{Z_2}\right]\right\}$$

$$l_0 = is_0(c_0^4 - a) + b$$

on the rib surface, and the homogeneous boundary-contact conditions (1.2). We introduce in the analysis the plate displacements $\beta(x)$ generated by the pressure field $w(x, y)$: $\beta(x) = w_y(x, 0)/(\rho_0\omega^2)$. These displacements satisfy the differential equation and boundary conditions

$$\beta''''(x) - k_0^4\beta(x) + \frac{1}{D}w(x, 0) = B\delta(x) + C\delta'(x) \qquad (-\infty < x < \infty) \tag{2.2}$$

$$Z_1\beta(0) - [\beta'''(0)] = 0, \; -Z_2\beta'(0) - [\beta''(0)] - \frac{1}{D}\int_0^h s\,[w(0, s)]\,ds = 0 \tag{2.3}$$

The pressure field $w(x, y)$ is represented in the form of the sum

$$w(x, y) = -\frac{i}{2}\rho_0 w^2\int_{-\infty}^{\infty}H_0^{(1)}\left(k\sqrt{(x-s)^2 + y^2}\right)\beta(s)\,ds + u(x, y) = \tag{2.4}$$

$$-\frac{i}{2}\rho_0\omega^2 H_0^{(1)} * \beta + u(x, y)$$

where the asterisk indicates the operation of convoluting functions [6].

As a result, for the plate displacements we obtain the integro-differential equation (convolution is calculated for $y = 0$)

$$\beta''''(x) - k_0^4\beta(x) - \frac{i}{2}q^5 H_0^{(1)} * \beta + \frac{1}{D}u(x, 0) = B\delta(x) + C\delta'(x) \tag{2.5}$$

$$(-\infty < x < \infty)$$

while in boundary conditions (2.3) the field $u(0, s)$ is substituted for the field $w(0, s)$. The pressure field $u(x, y)$ satisfies the Helmholtz equation with boundary conditions at the plate and rib

$$u_y(x, 0) = 0 \quad (-\infty < x < \infty) \tag{2.6}$$

$$u_x(0, y) + \rho_0\omega^2 y\beta'(0) = g_1(y) \quad (0 < y < h)$$

$$g_1(y) = g(y) + \frac{i}{2}\rho_0 w^2\frac{\partial}{\partial x}(H_0^{(1)} * \beta)(0, y)$$

We have thus obtained a combination of the integro-differential equation (2.5) for plate displacements $\beta(x)$ and of the boundary value problem (2.6) for the pressure field $u(x, y)$. These two problems are interconnected, since boundary conditions (2.6) contain the plate displacement term $\beta(x)$ and pressure appears in Eq. (2.5) as the load. Such reformulation of the input problem is convenient mainly because conditions (2.6) correspond to Neumann's problem for which it is possible to derive simple long-wave approximations.

3. It is possible to show that the field $u(x, y)$ is an odd function of the x- co-

ordinate. Representing it in the form of an expansion in terms of plane waves and taking into account the first of boundary conditions (2.6), we obtain

$$u(x, y) = \int_0^\infty p(\lambda) \cos \lambda y \exp[-\gamma(\lambda)|x|]\, d\lambda \operatorname{sign} x \tag{3.1}$$

Let us introduce in the anlaysis the pressure on the right-hand side of the rib surface

$$\psi(y) = u(+0, y) = \int_0^\infty p(\lambda) \cos \lambda y \, d\lambda \tag{3.2}$$

Using the previously mentioned oddness of field $u(x, y)$ we find that $\psi(y) = 0$ when $y > h$, hence

$$p(\lambda) = \frac{2}{\pi} \int_0^h \psi(s) \cos \lambda s \, ds \tag{3.3}$$

By formulating boundary conditions (2.6) in terms of function $p(\lambda)$ we obtain for these the dual integral equations

$$\int_0^\infty p(\lambda) \cos \lambda y \gamma(\lambda)\, d\lambda = \rho_0 \omega^2 y \beta'(0) - g_1(y) \quad (0 < y < h) \tag{3.4}$$

$$\int_0^\infty p(\lambda) \cos \lambda y \, d\lambda = 0 \quad (y > h)$$

that can be reduced to an integral equation of the second kind by setting $\gamma(\lambda) = \lambda + \varepsilon(\lambda)$ where $\varepsilon(\lambda) = (\lambda^2 - k^2)^{1/2} - \lambda$, and convert the operator that corresponds to the paired equations after the substitution of λ for $\gamma(\lambda)$ (the conversion formula can be found, e.g., in [8]). The final integral equation along the semiaxis $0 < \lambda < \infty$ is of the form

$$p(\lambda) = -h^2 \int_0^\infty p(\mu) \varepsilon(\mu) \int_0^1 s J_0(\lambda s h) J_0(\mu s h)\, ds d\mu - \tag{3.5}$$

$$\frac{2h^2}{\pi} \int_0^1 s J_0(\lambda s h) \int_0^1 \frac{1}{\sqrt{1-\eta^2}} [g_1(s\eta h) - \rho_0 \omega^2 \beta'(0) s\eta h]\, d\eta ds$$

where $J_0(z)$ is a Bessel function.

To obtain long-wave asymptotics it is more convenient to use the equation in terms of pressure $\psi(y)$ at the rib surface. For this we, first of all, write the Fourier transformation $N(\lambda)$ of function $\beta(x)$ in Eq. (2.5) in terms of function $\psi(y)$. Then, taking into account condtions (2.3) and formulas (3.1) and (3.3), we obtain

$$N(\lambda) = \int_{-\infty}^\infty \beta(x) \exp(-i\lambda x)\, dx = \frac{2i\lambda\gamma(\lambda)}{DL(\lambda)} \int_0^h \Phi(\lambda, s) \psi(s)\, ds \tag{3.6}$$

$$\Phi(\lambda, s) = \frac{1}{\gamma(\lambda)} \exp[-\gamma(\lambda)s] - \frac{1}{J_{2,1} - 2\pi/Z_2} \left[I(s) - \frac{2\pi}{Z_2} s\right]$$

$$I(s) = \int_{-\infty}^\infty \frac{\mu^2}{L(\mu)} \exp[-s\gamma(\mu)]\, d\mu$$

Using the theorem on Fourier transformation of convolution [6] from the last of formulas (2.6) we obtain

$$g_1(y) = g(y) + \rho_0\omega^2 \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{i\lambda}{\gamma(\lambda)} \exp[-\gamma(\lambda) y] N(\lambda) d\lambda \tag{3.7}$$

Finally, applying the cosine transformation to the integral equation (3.5) and taking into account (3.2), (3.6), and (3.7) we obtain the required equation

$$\psi(y) = \int_0^h K(y, s)\psi(s) ds + \psi_0(y) \quad (0 < y < h) \tag{3.8}$$

$$K(y, s) = -\frac{2h^2}{\pi}\int_0^\infty \varepsilon(\mu)\cos\mu s \int_{y/h}^1 \frac{\eta}{\sqrt{h^2\eta^2 - y^2}} J_0(\mu\eta h)\, d\eta d\mu +$$

$$\frac{2h^2q^5}{\pi^2}\int_{y/h}^1 \frac{\eta}{\sqrt{h^2\eta^2-y^2}}\int_0^1 \frac{1}{\sqrt{1-\tau^2}}\left\{\int_{-\infty}^{\infty} \frac{\lambda^2}{L(\lambda)}\exp[-\gamma(\lambda)\eta\tau h] \times\right.$$

$$\left.\Phi(\lambda, s)\, d\lambda - \frac{2\pi/Z_2}{J_{2,1} - 2\pi/Z_2}\eta\tau h[-I(s) + sJ_{2,1}]\right\} d\tau d\eta$$

$$\psi_0(y) = -\frac{2h^2}{\pi}\int_{y/h}^1 \frac{\eta}{\sqrt{h^2\eta^2-y^2}}\int_0^1 \frac{1}{\sqrt{1-\tau^2}} g(\eta\tau h)\, d\tau d\eta$$

Inspection of the integral equation (3.8) in the space of continuous functions $C[0, h]$ will show that when $\varepsilon \to 0$ $(\varepsilon = kh)$ the norm of the related integral operator is of order $O(\varepsilon^2 \ln\varepsilon)$. This implies that in the case of a rib of fairly small wave dimensions the principle of contractive mappings is applicable to Eq. (3.8). Note that the estimates obtained in [9] in connection with an investigation of long wave diffraction on a strip were used for proving the smallness of the norm.

4. Equation (3.8) is exact and makes possible numerical calculation of the diffraction field. The expression for such field in terms of function $\psi(y)$ is

$$Q(x, y) = Q_0(x, y) + \frac{1}{\pi}\int_0^h \psi(s)\left\{-q^5\int_{-\infty}^{\infty}\exp[i\lambda x - \gamma(\lambda)y] \times\right. \tag{4.1}$$

$$\left.\frac{i\lambda}{L(\lambda)}\Phi(\lambda, s)\, d\lambda + 2\int_0^\infty \cos\lambda y\cos\lambda s\exp[-\gamma(\lambda)|x|]\, d\lambda\, \mathrm{sign}\, x\right\} ds$$

We restrict the derivation of the approximate solution of Eq. (3.8) to the first iteration

$$\psi(y) = \psi_0(y) + \int_0^h K(y, s)\psi_0(s) ds + O(\varepsilon^4\ln\varepsilon)$$

For calculating the integrals appearing there we use the asymptotic expansions

$$I(s) = k^{-2}[i_{2,0} - si_{2,1} + s^2 d - s^2\ln s + O(s^3)] \tag{4.2}$$

$$i_{m,n} = k^{-m-n+4}J_{m,n}$$

$$2d = -i_{2,0} + \pi i + ai_{0,0} + bi_{0,-1} - 2\ln\gamma + 2\ln 2 + 3$$

$$\int_0^1 \sqrt{1-\tau^2}\int_0^\infty (\sqrt{\mu^2-1}-\mu)\cos\varepsilon\mu\tau \int_t^1 \frac{\eta}{\sqrt{\eta^2-t^2}} J_0(\varepsilon\mu\eta)\,d\eta d\mu d\tau = \tag{4.3}$$
$$\frac{\pi}{2}\sqrt{1-t^2}\left[-i\frac{\pi}{8}+\frac{1}{4}\ln\frac{\gamma\varepsilon}{4}+\frac{2t^2-5}{24}+O(\varepsilon^2\ln\varepsilon)\right]$$

where $\ln\gamma$ is the Euler's constant and $s\to 0$.

Formula (4.2) is based on the readily verified relation

$$k^2[I''(s)+I(s)] = \pi i H_0^{(1)}(s) + a i_{0,0} + b i_{0,-1} + O(s)$$

To prove formula (4.3) it is sufficient to expand the integral

$$\int_0^\infty (\sqrt{\mu^2-1}-\mu) J_0(\varepsilon\mu\tau) J_0(\varepsilon\mu\eta)\,d\mu$$

in series in the small parameter ε.

After the substitution for the Bessel function of its integral representation we have to expand the integral

$$\int_0^\infty (\sqrt{\mu^2-1}-\mu)\exp(i\varepsilon\mu m)\,d\mu$$

where m is a bounded quantity. This expansion can be obtained by introducing the substitution $\mu = \operatorname{ch} z$ and equating the obtained expression to the integral representation of Henkel's function.

Omitting lengthy intermediate calculations, we present only the final results. The pressure on the rib surface is of the form

$$w(\pm 0, y) = \pm\frac{2\varepsilon}{\pi} f\left(\frac{y}{h}\right), \quad f(t) = \pi\sqrt{1-t^2}\frac{c_0 s_0}{l_0}\times \tag{4.4}$$
$$\left\{-\frac{4b\varepsilon}{\zeta_2}\frac{1}{i_{2,1}-2\pi/\zeta_2}\left(1+\frac{t^2}{\sqrt{1-t^2}}\ln\frac{1+\sqrt{1-t^2}}{t}\right)-\varepsilon^2 S_0\times\right.$$
$$\left[\left((c_0^4-a)s_0^2+2b\frac{d-\ln{}^1\!/_2\varepsilon}{i_{2,1}-2\pi/\zeta_2}\right)\frac{1+2t^2}{12}-\frac{b}{i_{2,1}-2\pi/\zeta_2}\times\right.$$
$$\left(\frac{1-4t^2}{36}+\frac{t^3}{3}\frac{\arccos t}{\sqrt{1-t^2}}\right)+S_0\left(-\frac{i\pi}{8}+\frac{1}{4}\ln\frac{\gamma\varepsilon}{4}+\frac{2t^2-5}{24}+\right.$$
$$\left.\left.\left.\frac{b}{4}\left(\frac{i_{2,0}^2}{i_{2,1}-2\pi/\zeta_2}-i_{2,-1}\right)\right)\right]+O(\varepsilon^3\ln\varepsilon)\right\}$$

$$S_0 = c_0^4 - a - b\frac{i_{2,0}}{i_{2,1}-2\pi/\zeta_2}, \quad \zeta_1 = \frac{Z_1}{k^3}, \quad \zeta_2 = \frac{Z_2}{k}$$

which conforms to Meixner's condition "at the rib" [7]. Similar formulas for pressure were obtained in the case of wave diffraction on a rigid or elastic strip [10].

At considerable distances from the plate the field $Q(x, y)$ acquires the characteristics of a divergent cylindrical wave. Applying to the integrals in formulas (1.4) and (4.1) the method of steepest descent we obtain

$$Q(x,y) \sim \sqrt{\frac{2\pi}{kR}}\exp\left[i\left(kR-\frac{\pi}{4}\right)\right]\Psi(\varphi,\varphi_0), \quad (kR = k\sqrt{x^2+y^2} \gg 1)$$

where the function $\Psi(\varphi, \varphi_0)$ may be considered as the representation of the directionality pattern of the field dissipated in the fluid and φ is the angle of observation taken from the semiaxis $x > 0$

$$\Psi(\varphi, \varphi_0) = \Psi_0(\varphi, \varphi_0) - \frac{iss_0cc_0\varepsilon^2}{2ll_0}\Big\{SS_0 - \frac{\varepsilon^2}{4} \times \tag{4.5}$$
$$\Big\{b\frac{d - \ln {}^1\!/_2\varepsilon - {}^1\!/_4 - 64/(3\varepsilon\zeta_2)}{i_{2,1} - 2\pi/\zeta_2}(S + S_0) + \frac{1}{2}[Ss_0^2(c_0^4 - a) +$$
$$S_0s^2(c^4 - a)] + SS_0\Big[-\frac{3}{4} - \frac{i\pi}{2} + \ln\frac{\gamma\varepsilon}{4} +$$
$$b\Big(\frac{i_{2,0}^2}{i_{2,1} - 2\pi/\zeta_2} - i_{2,-1}\Big)\Big]\Big\} + O(\varepsilon^3 \ln \varepsilon)\Big\}$$
$$\Psi_0(\varphi, \varphi_0) = \frac{iss_0}{ll_0}\Big\{\frac{1}{i_{0,1} - 2\pi/\zeta_1} + \frac{cc_0}{i_{2,1} - 2\pi/\zeta_2}\Big\}2b$$

where the quantities s, c, l, and S are obtained from s_0, c_0, l_0, and S_0 by the substitution of φ for φ_0 .

5. The diffraction field directionality patterns in terms of energy $E(\varphi, \varphi_0) = |\Psi(\varphi, \varphi_0)|^2$ were calculated by formula (4.5). It was assumed that the steel plate and the steel stiffening rib were both of the same thickness $H_1 = H$ and were immersed in water. The dimensionless parameter $\delta = kH$ was varied from 0.02 to 0.06 in steps of 0.005. In Fig. 1 the solid line represents the pattern for $\delta = 0.06$, $\varepsilon = 0.6$, and $\varphi_0 = 30°$, while the dashed line shows the pattern calculated in the zero approximation, i.e. without allowance for sound reflection from the rib surface $E_0(\varphi, \varphi_0) = |\Psi_0(\varphi, \varphi_0)|^2$.

Allowance for reflection (from the rib) results in the increase of power emitted into the fluid and in the change of direction of maximum emission by a certain angle $\Delta\varphi$. Values of angle $\Delta\varphi$ and of the ratio $m = \max_\varphi E/\max_\varphi E_0$ of the indicated above patterns maxima for plates of various demensionless thickness δ at incidence angles φ_0 equal $10°$, $30°$, and $60°$ are tabulated below in columns A, B, and C, respectively. The dashes relate to angles $\Delta\varphi < 10°$ and ratios $m < 1.1$. In these calculations the dimensionless rib height was assumed to be $\varepsilon = 10\delta$. Hence in the case of fairly thin plates ($\delta \leqslant 0.02$ and $\varepsilon \leqslant 10\,\delta$) the allowance for sound reflection from the rib surface does not significantly affect the field.

It is interesting to investigate the power N emitted by the considered structure into the fluid

$$N(\varphi_0) = \int_0^\pi E(\varphi, \varphi_0)\,d\varphi, \quad N_0(\varphi_0) = \int_0^\pi E_0(\varphi, \varphi_0)\,d\varphi$$

Values of calculated $N(\varphi_0)$ are shown in Fig. 2, where curves 1, 2, and 3 correspond to $\delta = 0.04, 0.05$, and 0.06 , respectively. Solid curves relate to ribs of height $\varepsilon = 10\,\delta$ and the dash lines to $\varepsilon = 7.5\,\delta$. The dependence of quantity $\theta = 10 \lg (N / N_0)$ which defines the error introduced in the determination of power by the calculation of diffraction effects in the zero approximation, on the incidence angle is shown in Fig. 3. The numbering of these curves is consistent with the numbering in Fig. 2. The error of the approximate analysis increases with increasing parameter δ and with the

Table 1

$\delta \cdot 10^2$	A		B		C	
	$\Delta\varphi$	m	$\Delta\varphi$	m	$\Delta\varphi$	m
2	10°	—	10°	—	—	—
3	20°	1.29	15°	1.22	15°	—
4	20°	1.96	25°	1.78	25°	1.31
5	30°	2.52	30°	2.24	30°	1.48
6	40°	2.97	40°	2.57	30°	1.60

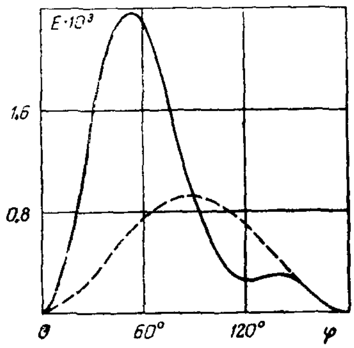

Fig. 1

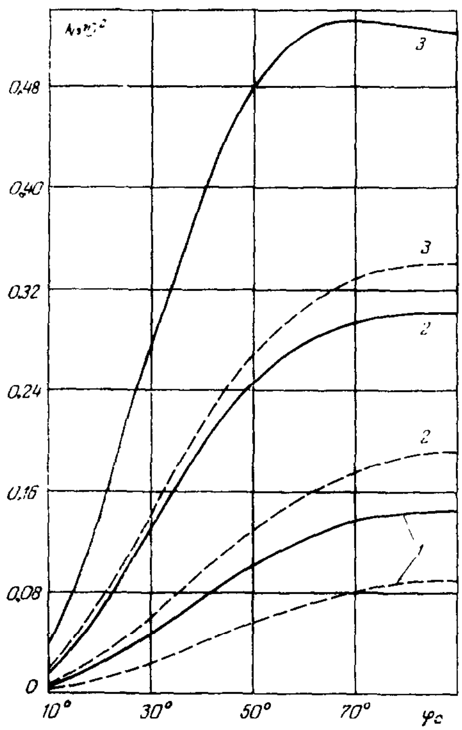

Fig. 2

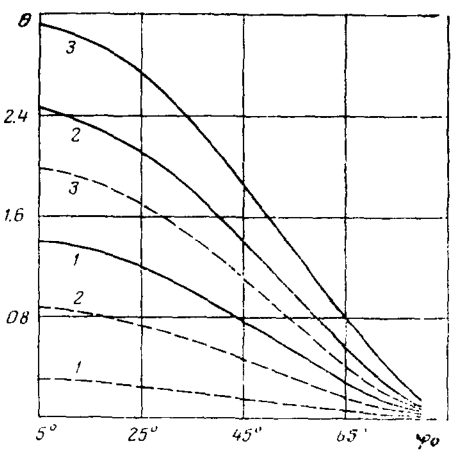

Fig. 3

incidence approaching the glancing angle.

Note that in the considered low-frequency range the reflection of sound from the rib surface results not only in an increase of the power emitted by the plate but, also, in its redistribution with respect to angles of emission.

The diffraction of a plane wave by a plate stiffened by a rib and surrounded by fluid on both sides can be investigated in the same manner.

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Translated by J. J. D.